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PROPERTIES OF INTUITIONISTIC (T, S) NORMED FUZZY IDEALS IN SEMIGROUPS

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Abstract: In this paper, we introduce the notation of intuitionistic (T, S) normed fuzzy ideals in a semigroup and investigate some properties.

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1. Introduction

The notation of a fuzzy set was introduced by L. A. Zadeh [9] and since then this concept has been applied to various algebraic structures. The concept of intuitionistic fuzzy set was introduced by K. T. Atanassov [2,3], as a generalization of the notation of fuzzy sets N. Kuroki [4] discussed characterizations of semigroups. K. H. Kim and Y. B. Jun [6] considered the intuitionistic fuzzification of the notation of several ideals in a semigroup and investigated some properties of such ideals. M. T. Abu Osman [1] defined t-norm T and Y. Yu, J. N. Mordeson and S. C. Cheng [7] defined s-norm S. In this paper, using (T, S)-norm we study intuitionistic fuzzy ideals of semigroups and establish some results.

2. Preliminaries

Definition 2.1. [6] *Let S be a semigroup. By a subsemigroup of S, we mean a non-empty subset A of S such that $A^2 \subseteq A$.*

Definition 2.2. [6] *By a left (right) ideal, we mean a non-empty subset A of S* *such that $SA \subseteq A$ ($AS \subseteq A$).*

Definition 2.3. [6] *A subsemigroup A of a semigroup S is called an interior ideal of S if $SAS \subseteq A$.*

Definition 2.4. [6] *By a two sided ideal or simply ideal, we mean a non-empty subset of S which is both a left and a right ideal of S.*

Definition 2.5. [6] *A subsemigroup of a semigroup S is called a bi-ideal of S if $ASA \subseteq A$.*

Definition 2.6. [6] *A semigroup S is said to be regular if for each $x \in S$, there exist $s \in S$ such that $x = xsx$.*

Definition 2.7. [6] *A semigroup S is said to be left (right) simple if S has no proper left (right) ideals.*

Definition 2.8. [6] *A semigroup S is called left-zero (right-zero) if $xy = x$ ($xy = y$) for all $x, y \in S$.*

Definition 2.9. [6] *If a semigroup S has no proper ideals, then we say that S is simple.*

Definition 2.10. [6] *An element e in a semigroup is called an idempotent if $ee = e$ and E_s denote the set of all idempotents in a semigroup S.*

Definition 2.11. [2] *A fuzzy subset μ of a semigroup S is called a fuzzy subsemigroup of S if $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$ for all $x, y \in S$.*

Definition 2.12. [2] *A fuzzy subsemigroup μ of a semigroup S is said to be a fuzzy bi-ideal of S if $\mu(xwy) \geq \min\{\mu(x), \mu(y)\}$ for all $x, w, y \in S$.*

Definition 2.13. [2] *A fuzzy subset μ of a semigroup S is called a fuzzy left ideal of S if $\mu(xy) \geq \mu(y)$ for all $x, y \in S$ (or μ is called a fuzzy right ideal if $\mu(xy) \geq \mu(x)$ for all $x, y \in S$).*

Definition 2.14. [1] *By a t-norm T, we mean a function $T : [0,1] \times [0,1] \to [0,1]$ satisfying the following conditions:*

$$
\begin{aligned}
&(1)\ T(x,1) = x,\\
&(2)\ T(x,y) \leq T(x,z)\ if\ y \leq z,\\
&(3)\ T(x,y) = T(y,x),\\
&(4)\ T(x,T(y,z)) = T(T(x,y),z),\ for\ all\ x,y,z \in [0,1].
\end{aligned}
$$

Proposition 2.15. [1] *Every t-norm T has a useful property: $T(\alpha, \beta) \leq \min(\alpha, \beta)$, for all $\alpha, \beta \in [0,1]$.*

Definition 2.16. [7] *By a s-norm S, we mean a function* $S : [0,1] \times [0,1] \to [0,1]$ *satisfying the following conditions:*

$$\begin{aligned}
&(1)\ S(x,0) = x,\\
&(2)\ S(x,y) \le S(x,z)\ if\ y \le z,\\
&(3)\ S(x,y) = S(y,x),\\
&(4)\ S(x,S(y,z)) = S(S(x,y),z),\ for\ all\ x,y,z \in [0,1].
\end{aligned}$$

Proposition 2.17. [7] *Every s-norm S has a useful property:* $S(\alpha,\beta) \ge \max(\alpha,\beta)$, *for all* $\alpha,\beta \in [0,1]$.

3. Intuitionistic fuzzy ideals

In what follows, let S denote a semigroup unless otherwise specified.

Definition 3.1. *An IFS A in S is called an intuitionistic (T,S)-normed fuzzy sub-semigroup of S if*

$$\begin{aligned}
&(1)\ \mu_A(xy) \ge T\{\mu_A(x), \mu_A(y)\},\\
&(2)\ \nu_A(xy) \le S\{\nu_A(x), \nu_A(y)\},\ for\ all\ x,y \in S.
\end{aligned}$$

Definition 3.2. *An IFS A in S is called an intuitionistic fuzzy left ideal of S if* $\mu_A(xy) \ge \mu_A(y)$ *and*$\nu_A(xy) \le \nu_A(y)$, *for all* $x,y \in S$.

Definition 3.3. *An IFS A in S is called an intuitionistic fuzzy right ideal of S if* $\mu_A(xy) \ge \mu_A(x)$ *and*$\nu_A(xy) \le \nu_A(x)$, *for all* $x,y \in S$.

Definition 3.4. *An IFS A in S is called an intuitionistic fuzzy ideal of S if it is both an intuitionistic fuzzy right ideal and an intuitionistic fuzzy left ideal of S.*

Definition 3.5. *An intuitionistic (T, S)-normed fuzzy sub-semigroup A of S is called an intuitionistic (T, S)-normed fuzzy bi-ideal of S if*

$$\begin{aligned}
&(1)\ \mu_A(xwy) \ge T\{\mu_A(x), \mu_A(y)\},\\
&(2)\ \nu_A(xwy) \le S\{\nu_A(x), \nu_A(y)\},\ for\ all\ x,w,y \in S.
\end{aligned}$$

Example 3.1. Let $S = \{a,b,c,d,e\}$ be a semigroup with the following cayley table:

.	*a*	*b*	*c*	*d*	*e*
a	*a*	*a*	*a*	*a*	*a*
b	*a*	*a*	*a*	*a*	*a*
c	*a*	*a*	*c*	*c*	*e*
d	*a*	*a*	*c*	*d*	*e*
e	*a*	*a*	*c*	*c*	*e*

Let $T : [0, 1] \times [0, 1]$ be a function defined by $T(\alpha, \beta) = \max(\alpha+\beta-1, 0)$, for all $\alpha, \beta \in [0, 1]$ and $S : [0, 1]\times[0, 1]$ be a function defined by $S(\alpha, \beta) = \min(\alpha+\beta, 1)$, for all $\alpha, \beta \in [0, 1]$. Then T is a t-norm and S is a s-norm. Define an IFS $A =< \mu_A, \nu_A >$ in S by $\mu_A(a) = 0.6,\ \mu_A(b) = 0.5,\ \mu_A(c) = 0.4,\ \mu_A(d) = \mu_A(e) = 0.3,\ \nu_A(a) = \nu_A(b) = 0.3,\ \nu_A(c) = 0.4,\ \nu_A(d) = 0.5$ *and* $\nu_A(e) = 0.6$.

By routine calculation we can check that $A =< \mu_A, \nu_A >$ is an intuitionistic (T, S)-normed fuzzy bi-ideal of S.

Theorem 3.6. *If A is an intuitionistic (T, S)-normed fuzzy subsemigroup of S, then* $\bar{A} =< \chi_A, \bar{\chi}_A >$ *is an intuitionistic (T, S)-normed fuzzy subsemigroup of S.*

Proof. Let $A =< \mu_A, \nu_A >$ be an intuitionistic (T, S)-normed fuzzy subsemigroup of S and let $x, y \in S$. Then $\mu_A(xy) \geq T\{\mu_A(x), \mu_A(y)\}$ and $\nu_A(xy) \leq S\{\nu_A(x), \nu_A(y)\}$.

If $x, y \in A$, then $xy \in A$. In this case $\chi_A(xy) = 1 \geq T\{\chi_A(x), \chi_A(y)\}$

$$\begin{aligned}\bar{\chi}_A(xy) =& 1 - \chi_A(xy)\\ \leq& 1 - T\{\chi_A(x), \chi_A(y)\}\\ =& S\{1 - \chi_A(x), 1 - \chi_A(y)\} \ = S\{\bar{\chi}_A(x), \bar{\chi}_A(y)\}.\end{aligned}$$

If $x \notin A$ or $y \notin A$, then $\chi_A(x) = 0$ or $\chi_A(y) = 0$. Thus

$$\begin{aligned}\chi_A(xy) \geq 0 =& T\{\chi_A(x), \chi_A(y)\} \text{ and}\\ S\{\bar{\chi}_A(x), \bar{\chi}_A(y)\} =& S\{1 - \chi_A(x), 1 - \chi_A(y)\}\\ =& 1 - T\{\chi_A(x), \chi_A(y)\} \ = 1 \geq \bar{\chi}_A(xy).\end{aligned}$$

Hence $\bar{A} =< \chi_A, \bar{\chi}_A >$ is an intuitionistic $(T,\ S)$-normed fuzzy subsemigroup of S.

Theorem 3.7. *Let A be a non-empty subset of S. If* $\bar{A} =< \chi_A, \bar{\chi}_A >$ *is an intuitionistic (T, S)-normed fuzzy subsemigroup of S, then A is subsemigroup of S.*

Proof. Suppose that $\bar{A} =< \chi_A, \bar{\chi}_A >$ is an intuitionistic (T, S)-normed fuzzy subsemigroup of S and $x \in A^2$. In this case $x = uv$ for some $u, v \in A$. Now $\chi_A(x) = \chi_A(uv) \geq T\{\chi_A(u), \chi_A(v)\} = 1$.

Hence $\chi_A(x) = 1$, that is $x \in A$. Thus A is subsemigroup of S.

$$\begin{aligned}\bar{\chi}_A(x) = \bar{\chi}_A(uv) \leq& S\{\bar{\chi}_A(u), \bar{\chi}_A(v)\}\\ =& S\{1 - \chi_A(u), 1 - \chi_A(v)\} = 0 \text{ and so}\\ \bar{\chi}_A(x) =& 1 - \chi_A(x) = 0.\end{aligned}$$

Therefore $\chi_A(x) = 1$, that is $x \in A$. Hence A is subsemigroup of S.

Theorem 3.8. *Let U be a left-zero subsemigroup of S. If $\bar{A} =< \mu_A, \nu_A >$ is an intuitionistic fuzzy left ideal of S, then the restriction of A to U is constant, that is, $A(x) = A(y)$ for all $x, y \in U$.*
Proof. Let $x, y \in U$. Since U is a left-zero, $xy = x$ and $yx = y$. In this case, we have

$$\begin{aligned}\mu_A(x) =&\mu_A(xy) \geq \mu_A(y),\ \mu_A(y) = \mu_A(yx) \geq \mu_A(x),\\ \nu_A(x) =&\nu_A(xy) \leq \nu_A(y) \text{ and } \nu_{\mathrm{A}}(\mathrm{y}) = \nu_{\mathrm{A}}(\mathrm{yx}) \leq \nu_{\mathrm{A}}(\mathrm{x}).\end{aligned}$$

Thus we obtain $\mu_A(x) = \mu_A(y)$ and $\nu_A(x) = \nu_A(y)$, for all $x, y \in U$.
Hence $A(x) = A(y)$, for all $x, y \in U$.

Lemma 3.9. *If A is a left ideal of S, then $\bar{A} =< \chi_A, \bar{\chi}_A >$ is an intuitionistic fuzzy left ideal of S.*
Proof. Let $x, y \in S$. Since A is a left ideal of S, $xy \in A$ if $y \in A$. It follows that

$$\begin{aligned}\chi_A(xy) =&1 = \chi_A(y) \text{ and}\\ \bar{\chi}_A(xy) =&1 - \chi_A(xy) = 0 = 1 - \chi_A(y) = \bar{\chi}_A(y).\end{aligned}$$

If $y \notin A$, then $\chi_A(y) = 0$. In this case

$$\begin{aligned}\chi_A(xy) \geq&0 = \chi_A(y) \text{ and}\\ \bar{\chi}_A(y) =&1 - \chi_A(y) = 1 \geq \bar{\chi}_A(xy).\end{aligned}$$

Consequently, $\bar{A} =< \chi_A, \bar{\chi}_A >$ is an intuitionistic fuzzy left ideal of S.

Theorem 3.10. *Let $A =< \mu_A, \nu_A >$ be an intuitionistic fuzzy left ideal of S. If E_s is a left-zero subsemigroup of S, then $A(e) = A(e')$ for all $e, e' \in E_s$.*
Proof. Let $e, e' \in E_s$. Now $ee' = e$ and $e'e = e'$. Thus, since $A =< \mu_A, \nu_A >$ is an intuitionistic fuzzy left ideal of S, we get that

$$\begin{aligned}\mu_A(e) =&\mu_A(ee') \geq \mu_A(e'),\\ \mu_A(e') =&\mu_A(e'e) \geq \mu_A(e),\\ \nu_A(e) =&\nu_A(ee') \leq \nu_A(e') \text{ and} \nu_{\mathrm{A}}(\mathrm{e'}) = \nu_{\mathrm{A}}(\mathrm{e'e}) \leq \nu_{\mathrm{A}}(\mathrm{e}).\end{aligned}$$

Hence we have $\mu_A(e) = \mu_A(e')$ and $\nu_A(e) = \nu_A(e')$, for all $e, e' \in E_s$. This completes the proof.

Definition 3.11. *An IFS $A =< \mu_A, \nu_A >$ in S is said to be an intuitionistic fuzzy interior ideal if*

$$\begin{aligned}&(1)\ \mu_A(xsy) \geq \mu_A(s)\\ &(2)\ \nu_A(xsy) \leq \nu_A(S)\ for\ all\ s, x, y \in S.\end{aligned}$$

Remark 3.12. *It is clear that every intuitionistic fuzzy ideal of S is an intuitionistic fuzzy interior ideal of S.*

Theorem 3.13. *If S is regular, then every intuitionistic fuzzy interior ideal of S is an intuitionistic fuzzy ideal of S.*

Proof. Let $A =< \mu_A, \nu_A >$ be an intuitionistic fuzzy interior ideal of S and $x, y \in S$. In this case, because S is regular, there exist $s, s' \in S$ such that $x = xsx$ and $y = ys'y$. Thus

$$\begin{aligned} \mu_A(xy) =& \mu_A(x(ys'y)) \ = \mu_A(xy(s'y)) \geq \mu_A(y) \text{ and} \\ \nu_A(xy) =& \nu_A(x(ys'y)) \ = \nu_A(xy(s'y)) \ \leq \nu_A(y) \end{aligned}$$

It follows that $A =< \mu_A, \nu_A >$ is an intuitionistic fuzzy left ideal of S. Similarly we can show that $A =< \mu_A, \nu_A >$ is an intuitionistic fuzzy right ideal of S. This completes the proof.

Theorem 3.14. *Let S be a regular and let A be a non-empty subset of S. If $\bar{A} =< \chi_A, \bar{\chi}_A >$ is an intuitionistic fuzzy interior ideal of S, then A is an interior ideal of S.*

Proof. By Theorem 3.7, A is a subsemigroup of S. Suppose that $\bar{A} =< \chi_A, \bar{\chi}_A >$ is an intuitionistic fuzzy interior ideal of S and $x \in SAS$. In this case, $x = sat$, for some $s, t \in S$ and $a \in A$. It follows that $\chi_A(x) = \chi_A(sat) \geq \chi_A(a) = 1$. Hence $\chi_A(x) = 1$, that is $x \in A$.

$$\begin{aligned} \bar{\chi}_A(x) =& \bar{\chi}_A(sat) \leq \bar{\chi}_A(a) = 1 - \chi_A(a) = 0, \\ \bar{\chi}_A(x) =& 1 - \chi_A(x) = 0. \end{aligned}$$

Therefore $\chi_A(x) = 1$, that is $x \in A$. Thus A is a interior ideal of S.

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